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Generalized Nonequilibrium Binary Scaling for Shock Standoff on Hypersonic Blunt Bodies

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Introduction

THE shock standoff distance δ_s in a blunt-body stagnation region (Fig. 1) is sensitive to the thermochemistry within the shock layer[1] and, hence, an important observable in hypervelocity test facilities.[2] Although numerical codes are available to predict δ_s, they are expensive for engineering parametric studies and do not yield physical insight and similitude laws needed for experimental design and data interpretation. On the other hand, existing analytical methods may not permit extension to include multitemperature ionization. The present paper examines a new analytical theory[3] of shock standoff with a nonequilibrium-dissociated shock layer to demonstrate a generalized binary scaling property for high-altitude hypervelocity flight simulation work.

Theoretical Formulation

We consider a blunt nose region at zero angle of attack under the following assumptions (Fig. 1): 1) The postshock static pressure is a known constant across the shock layer. 2) The tangential velocity component is of the form $U \approx \beta_S x$, where β_S is an appropriately defined known constant equal to the effective stagnation point velocity gradient reflecting the $U(y)$ variation. 3) Low Reynolds number viscous shock layer effects are negligible. For shock layer Reynolds numbers above 300 (pertaining to many applications), these assumptions are sufficient to model the main aerothermochemical aspects of the flow along the stagnation line $x \to 0$.

Regardless of the gas or its chemistry, continuity yields the normal velocity component V as

$$\rho(y)V(y) = -(1+J)\beta_S \int_0^y \rho \, \mathrm{d}y \tag{1}$$

where $J = 0, 1$ for two-dimensional or axisymmetric flow, respectively. When the density-stretched coordinate η is introduced,

$$\eta \equiv (1+J)\frac{\beta_S R_B}{U_\infty} \int_0^y \frac{\rho}{\rho_\infty} \mathrm{d}\left(\frac{y}{R_B}\right) \tag{2}$$

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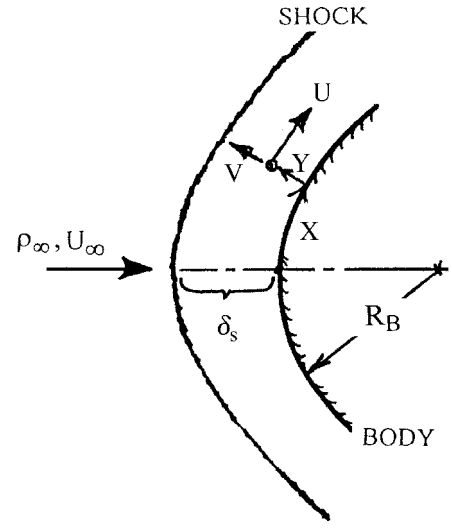

Fig. 1 Schematic of blunt-body stagnation shock layer.

Eq. (1) yields simply $\rho V = -\rho_\infty U_\infty \eta$. Because $\rho V = -\rho_\infty U_\infty$ behind the shock, $\eta = 1$ is, thus, the shock location regardless of the flow chemistry or dimensionality. The physical standoff distance $\delta_S = (y)_{\eta=1}$ by Eq. (2) is then

$$\frac{\delta_S}{R_B} = \left(\frac{U_\infty}{\beta_S R_B}\right)\left(\int_0^1 \left(\frac{\rho_\infty}{\rho}\right)\mathrm{d}\eta \Big/ (1+J)\right) \tag{3}$$

Equation (3) shows that once the density profile $\rho(\eta)$ is found using the species, energy, and state equations, one integration yields δ_s in a convenient nondimensional form.

For dissociation involving the net formation of an atom mass fraction α, we have along the stagnation line (neglecting diffusion) that $\rho V \, \mathrm{d}\alpha/\mathrm{d}y = \dot{w}_\alpha$, where $\dot{w}_\alpha$ is the net chemical rate of atom mass formation per unit volume. Then applying Eq. (2), we obtain the nondimensional equation

$$\eta\frac{\mathrm{d}\alpha}{\mathrm{d}\eta} = \frac{-(\dot{w}_\alpha/\rho)}{(1+J)\beta_S} = \frac{-(R_B U_\infty)(\dot{w}_\alpha \rho)}{(1+J)(\beta_S R_B/U_\infty)} \tag{4}$$

where both $\dot{w}_a/\rho$ and U_∞/R_B have the same units of reciprocal time.

Application to Dissociating Diatomic Gases

We now consider the specific case of a diatomic gas that undergoes the dissociation–recombination reaction

$$A_2 + M \underset{k_R}{\overset{k_D}{\rightleftarrows}} 2A + M \tag{5}$$

where M is a third body (molecule A_2 or atom A), k_D is the dissociation rate, and k_R is the recombination rate. The corresponding $\dot{w}_\alpha$ is formulated from the law of mass action combined with the principle of detailed balancing and the use of the usual mole-mass fraction relationship, giving Eq. (4) as[3]

$$\eta\frac{\mathrm{d}\alpha}{\mathrm{d}\eta} = \frac{-k_{DA2}\rho_\infty R_B U_\infty}{2(1+J)R_u T\lambda}\sqrt{\frac{(1-\varepsilon_F)}{2\varepsilon_F}}\left\{1+\left[2\left(\frac{k_{DA}}{k_{DA2}}\right)-1\right]\alpha\right\}$$

$$\times(1+\alpha)^{-1}\left\{(1-\alpha)-\left(\frac{4\rho R_{A2}T}{K_{\mathrm{eq}}}\right)\alpha^2\right\} \tag{6}$$

where $\varepsilon_F = \rho_\infty/\rho_F$, $k_{DA2} \equiv CT^N e^{-T_D/T}$, c_j are mass fractions $[c_A = \alpha, \; c_{A2} = (1-\alpha)]$, and $K_{\mathrm{eq}} = p_{\mathrm{ref}}(T/T_{\mathrm{ref}})^s e^{-(T_D/T)}$ in terms of the parameters p_{ref}, T_{ref}, s, and the dissociation temperature T_D.

The density ρ can be expressed in terms of α, the pressure p, and the mixture temperature T by the equation of state, which on use of Dalton's law becomes

$$p = \rho(R_u/M_{A2})T(1+\alpha) \tag{7}$$

Here $p \approx \rho_\infty U_\infty^2(1-\varepsilon_F)$ and $\beta_S R_B/U_\infty \approx \sqrt{[2\varepsilon_F(1-\varepsilon_F)]}\lambda$ from constant density shock layer Newtonian theory (see Ref. 4), where $\lambda = k/\arcsin k$ with $k = \sqrt{(1-3\varepsilon_F)}/\sqrt{(3\varepsilon_F)}$ for $J=0$ and $\lambda = [1+\sqrt{(8\varepsilon_F/3)}]/\sqrt{(8\varepsilon_F/3)}$ for $J=1$. The corresponding energy conservation equation for steady, adiabatic, nonradiating shock layer (constant total enthalpy) at hypersonic flight conditions ($U_\infty^2 \gg R_{A_2}T_\infty$) is[3]

$$\alpha + \tfrac{5}{2}(T/T_D)\left[1+\alpha+\tfrac{2}{5}(1-\alpha)(J_V+1)\right] \approx \left(U_\infty^2/2h_D\right) + \alpha_\infty \tag{8}$$

where $h_D \equiv R_{A_2}T_D = R_uT_D/M_{A_2}$, M_{A_2} is the molecular weight and J_V is an index that indicates the fraction of classical vibrational internal molecular energy assumed to be activated, for example, $J_V = 1$ for complete activation, $J_V = 0$ for negligible vibration, and $J_V = \frac{1}{2}$ for the Lighthill ideal dissociating gas model.

Similitude Properties and Binary Scaling

The foregoing indicates that the shock properties obey the following general similitude law for dissociating diatomic gases: $\alpha(\eta), \theta(\eta), \ell(\eta) = f_{ns}[\Gamma_D, \tau_D, \alpha_\infty, (1-\varepsilon_F)\rho_\infty/\rho_{\text{ref}}]$, where $\theta \equiv T/T_D$, $\mathcal{I} \equiv \rho/\rho_\infty(1-\varepsilon_F)$ and $\tau_D \equiv U_\infty^2/2h_D$. Here $\rho_{\text{ref}} \equiv p_{\text{ref}}M_{A2}(T_D/T_{\text{ref}})^S/R_uT_D$, and Γ_D is the blunt nose Damköhler number

$$\Gamma_D \equiv \frac{CT_D^{N-1}R_B(1-\varepsilon_F)\rho_\infty U_\infty}{2(1+J)R_u\sqrt{2\varepsilon_F(1-\varepsilon_F)\lambda}} \tag{9}$$

The corresponding scaling law governing δ_s, from Eq. (3), is

$$(1+J)\frac{\delta_S}{R_B}(1-\varepsilon_F)\sqrt{2\varepsilon_F(1-\varepsilon_F)} = \int_0^1 \ell^{-1}(\eta)\,\mathrm{d}\eta$$

$$\equiv I\left[\Gamma_D, \tau_D, \alpha_\infty, \frac{(1-\varepsilon_F)\rho_\infty}{\rho_{\text{ref}}}\right] \tag{10}$$

when I is an integral function of the indicated scaling parameters and the last term on the right derives from the recombination term $\sim\alpha^2$ in Eq. (6).

The practical utility of Eq. (10) obtains in those high-altitude/hypervelocity regimes of flight where recombination is negligible and the last parameter drops out. Then the appropriately nondimensionalized standoff distance depends for a given α_∞ only on the Damköhler number Γ_D and the parameter τ_D, regardless of altitude ρ_∞, flight velocity, or body size. A particular version of this binary scaling for fixed U_∞ and $\rho_\infty \cdot R_B$ has in fact been used for some time[5,6] to simulate nonequilibrium flows. A second and newer aspect, which we will discuss, deals with the roles of the parameters α_∞ and (especially) τ_D. In particular, we examine when τ_D ceases to be influential and, hence, no longer restricts the usual binary scaling to require a fixed type of gas and/or flight velocity. Such a situation is important in hypervelocity testing because then the similitude requirements are unhooked from the need to duplicate τ_D and, hence, U_∞.

Parametric Study Results

Damköhler Number Correlation

Based on the chemical kinetic data of Park[7] and the assumption that $J_v = \frac{1}{2}$, Fig. 2 shows the Damköhler number effect in terms of the nondimensional standoff distance ratio $r_s \equiv (\delta_S - \delta_{S,\text{eq}})/(\delta_{S,F} - \delta_{S,\text{eq}})$, where the subscripts eq and F denote equilibrium and frozen shock layer values, respectively. These results are plotted vs the modified Damköhler number Ω (proportional to Γ_D) defined by $\Omega \equiv \theta_F^{N-1}e^{-1/\theta_F}(1-\alpha_\infty)\Gamma_D\{1+[2(k_{DA}/k_{DA2})-1]\alpha_\infty\}/(1+\alpha_\infty)$. When it is noted that r_S always lies between 0 in the equilibrium limit and unity in the opposite chemically frozen limit, this presentation efficiently collapses both theory and experiment to nearly universal sets of curves. Also shown is the envelope of combined experimental data/numerical computational fluid dynamic calculations.[8–10] It is seen that the present theory is in good agreement in predicting a significant decrease in r_S as Ω increases from the chemically frozen limit ($\Omega \to 0$, $r_S = 1$) to the equilibrium-dissociated limit ($\Omega \to \infty$, $r_S = 0$). The various analyses lie within the experimental accuracy ($\pm 10\%$) of the available data; indeed, the present theory lies near its center.

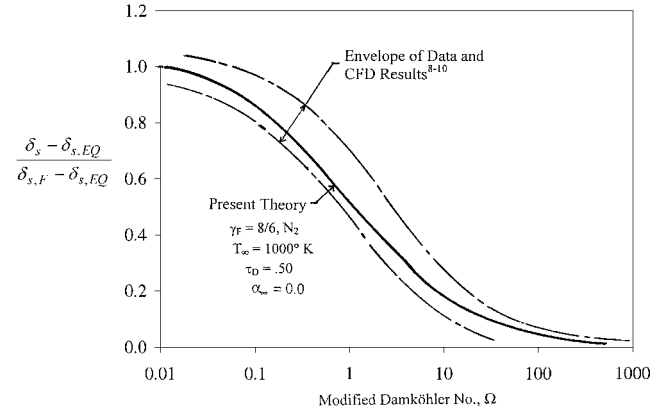

Fig. 2 Correlation of nondimensional shock standoff distance vs Damköhler number.

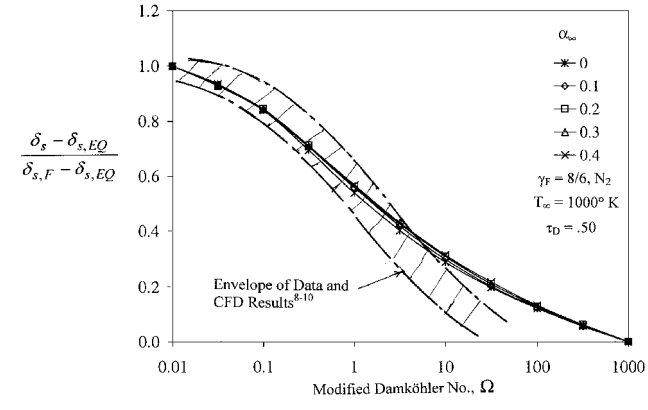

Fig. 3a Effect of freestream dissociation on the $r_s(\Omega)$ correlation.

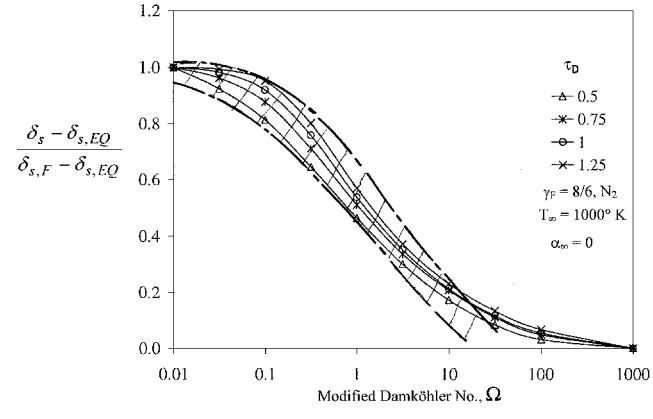

Fig. 3b Influence of the similitude parameter τ_D on the $r_s(\Omega)$ correlation.

Freestream Dissociation Effect

The effect of nonequilibrium freestream dissociation α_∞ on r_S over the entire nonequilibrium range is shown in Fig. 3a. For $0 \le \alpha_\infty \le 0.4$, it is seen that α_∞ has a negligible effect up to $\Omega \le 100$; within the experimental uncertainty, the α_∞-effect has, thus, been almost completely accounted for in the $\delta_{S,\text{eq}}$ and δ_{SF} values involved in r_S and its influence on Ω.

Nondimensional Kinetic Energy Parameter τ_D

The similitude parameter τ_D represents the ratio of the flight kinetic energy to the dissociation energy of the gas; its influence on

the r_S vs Ω variation is shown in Fig. 3b. For $\tau_D \ll 1$, it has a significant influence on r_S by decreasing the relative nonequilibrium effect with increasing τ_D. However, when $\tau_D \geq 1$, the r_S vs Ω correlation becomes insensitive to τ_D. Indeed, for $\tau_D \geq 1$, its parametric effect is far smaller than the typical experimental data uncertainty, and, thus, τ_D ceases to be relevant. In such cases, which embrace a wide range of planetary entry vehicle conditions, we, thus, obtain the generalized binary scaling that the ratio r_S depends only on Ω regardless of the specific values of ρ_∞, R_B, U_∞, and α_∞ or type of gas.

With regard to practical applications, the following simple closed-form expression has been found to fit the curve (Fig. 3) throughout the entire nonequilibrium regime:

$$r_s \approx \frac{6\sqrt{1+5.6\Omega}-1}{5+20\Omega} \tag{11}$$

Equation (11) yields the linear function $r_s \cong 1 - 0.70\Omega$ in the nearly frozen limit $\Omega \ll 1$, and the inverse square root $r_s \cong 0.70\Omega^{-1/2}$ for nearly equilibrium flow $\Omega \gg 1$.

Conclusion

We have presented some new parametric study results from a recent analytical theory of hypersonic blunt nose shocks standoff[3] which establish an extended nonequilibrium-dissociation binary scaling concept wherein the need to simulate flight velocity is eliminated when the parameter $\tau_D \equiv U_\infty^2/2hD$ is greater than unity.

Acknowledgments

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